Minimizing life cycle costs of industrial LV distribution networks
R. I. Senaratna and N. I. Senaratna

Abstract
Sizing of low voltage cables and positioning of distribution switchgear, in industrial and commercial end-user environments have been, in the main, based on installation environment, current carrying capacity and voltage drop considerations for cable sizing, and, operational convenience and aesthetic considerations for switchgear positioning. Except indirectly (as in the case of specifying the smallest cable with the appropriate current carrying capacity), cost considerations have rarely been factored into this decision process.

Perhaps, the first attempt in this direction in Sri Lanka was a paper presented by one of the authors at the First Annual IEE Sri Lanka Conference (1994) which demonstrated how distribution switchgear could be positioned in a cost-effective manner in three phase wiring networks typically found in industrial environments. However, this did not take into account voltage drop limits, nor energy losses in the wiring network over the life of the installation.

Ideally, optimal distribution switchgear locations and cable sizes should result in minimizing the sum of cable costs and the energy-loss costs subject to voltage drop and current carrying capacity constraints. The innate non-linearities in numerical relationships between factors and other non-continuous aspects of the underlying data make a solution using conventional methods difficult (if not impossible).

This paper proposes a Simulated Annealing based technique for finding distribution switchgear locations and cable sizes that minimize ‘life-cycle’ costs of a part of a three phase wiring network. Simulated Annealing is a variation of the “Hill Climbing” Algorithm and has, in the past, proved quite successful in solving similar problems.

By applying the technique to an example environment, we demonstrate that it results in significant cost reduction.

1. Introduction
The procedure that is normally followed in sizing a power cable is well established and is detailed in the literature [2] associated with the IEE Wiring Regulations [1]. Subject to voltage drop limits, this procedure would give the smallest cable size that could be safely used in a given environment. Whilst this approach minimizes the initial investment, it does not take into account energy losses that would occur during the life of a cable. It was generally assumed that such losses would be a very small fraction of the energy used. As a result, electrical engineers have had recourse to other methods of reducing the cost of wiring such as trying to minimize the total volume of copper in the cable network. A particularly interesting variant of this approach has revealed [3] that subject to certain restrictions, the minimal amount of copper is found when the voltage drop per unit length is the same throughout the distribution system. A year after the publication of Jones’s article, one of the authors of this paper tried a different approach [5] wherein it was demonstrated that judicious positioning of intermediate distribution switchgear would contribute to cost reduction.

However, the importance of economic optimization of power cable size was not lost on organizations responsible for developing engineering standards. Both the British Standards Institution and the International Electrotechnical Commission attempted to address this issue through their standards BS 7450: 1991 and IEC 1059: 1991-02 respectively. These documents approach the problem of cost minimization from the perspective of the providers of public HV distribution feeder systems. It was assumed that, to cater to load growth, the cables should initially have considerable excess capacity. It was also assumed that constraints associated with the positioning of intermediate distribution switchgear did not exist. Another negative feature was that the method described seemed incapable of dealing with sub-optimal local cost minima.

Rapid and sustained increases in the cost of fuels used to generate electricity have invalidated the assumption that energy losses in cables in industrial or commercial complexes are negligible. A simple calculation, with present day wiring costs and electricity prices as inputs, would reveal that the capitalized cost of energy losses over the life of an installation is anything but negligible. This is so even when cable sizes have to be increased to circumvent unacceptable voltage drops.

This paper demonstrates the use of an algorithm which, not only takes into account energy losses and the initial investment, but also factors in voltage drop considerations and the financial benefits that would accrue if items of intermediate distribution switchgear are optimally positioned.

2. Background
2.1. The problem
An electrical distribution network might be thought of as one single tree \( T(N, E) \) with edges directed from parent nodes to child nodes. The root of the tree is the main switchboard (MSB). The leaf nodes of the tree are the end-applications. There may be (in theory) any number of intermediate levels in the tree. However, very often, there is simply a single power distribution board (PDB) between the MSB and the appliances.

As mentioned, the nodes \( n_i \in N \) correspond to appliances, distribution boards and switchboards. Each node has, at most, one parent node \( \text{Parent}(n_i) \in N \) and a set of zero or more child nodes \( \text{ChildSet}(n_i) \subset N \). Each
node has a position relative to some reference point \((X(n_i), Y(n_i))\) in \(m\).

If the node is a leaf (an appliance), it has some current load. We denote the maximum current by \(I_{\text{max}}(n_i)\) and the average current by \(I_{\text{ave}}(n_i)\). In this paper we take, \[ I_{\text{ave}}(n_i) = \text{LoadFactor} \cdot I_{\text{max}}(n_i). \] A non-leaf node needs to support the load of all its child nodes summed. We take \(I_{\text{max}}(n_i)\) of a no-child node as the sum of the \(I_{\text{ave}}\) of its child nodes.

\[ I_{\text{ave}}(n_i) = \sum_{j \in \text{ChildSet}(n_i)} I_{\text{ave}}(n_j) \]  
...Eq. 1

Taking \(I_{\text{max}}(n_i)\) of a no-child node as the sum of the \(I_{\text{max}}\) of its child nodes is too conservative as precedence relationships and other factors result in all the child nodes (appliances) never operating at full load at the same time. Hence, we include a diversity factor (DiversityFactor) to take this into account.

\[ I_{\text{max}}(n_i) = \text{DiversityFactor} \cdot \sum_{j \in \text{ChildSet}(n_i)} I_{\text{max}}(n_j) \]  
...Eq. 2

In keeping with good installation practice in industrial settings, cables are run parallel to building lines. We also assume that the average vertical distance associated with a cable run is \(h\) and all cables would, therefore, have to travel a distance \(2h\) in the vertical (Z) direction. Hence the distance between any two nodes is:

\[ \text{Dis}(n_i, n_j) = |X(n_i) - X(n_j)| + |Y(n_i) - Y(n_j)| + 2h \]  
...Eq. 3

Since we consider only straight line distances, X and Y positions of nodes are independent, and hence might be considered separately in calculations. Some of the nodes will have fixed positions, while others might be varied within certain bounds. For the bulk of this paper, we will consider a system with one MSB, one PDB and \(k\) appliances, labelled \(n_0, n_{\text{PDB}}, n_1, n_2, \ldots, n_k\). The positions of \(n_0, n_1, \ldots, n_k\) are fixed; that of \(n_{\text{PDB}}\) is variable.

\[ X(n_{\text{PDB}}), Y(n_{\text{PDB}}) \]  
...Optimization Variable 1 (OV1)

The edges \((e_i \in E)\) of the tree correspond to cables. A cable could be one of several types as available on the market \((\text{cable}(e_i) = c_j)\) where \(c_j \in C\).

\[ \text{Cable}(e_i) \]  
...Optimization Variable 2 (OV2)

A cable is characterized by several properties: Cost per unit distance \((\text{Cost}(c_j)\) in Rs/\(m\)), current carrying capacity \((\text{I}(c_j)\) in \(A\)), resistance per unit distance \((\text{R}(c_j)\) in \(\Omega/\text{m}\)), and reactance per unit distance \((\text{D}(c_j)\) in \(\text{VA}^{-1}\text{m}^{-1}\)).

An edge is directed from a node to one of its child nodes, \(\text{Head}(e_i), \text{Tail}(e_i) \in N\) and \(\text{Tail}(e_i) \in \text{ChildSet}(\text{Head}(e_i))\). The length of an edge is \(\text{Len}(e_i) = \text{Dis}(\text{Head}(e_i), \text{Tail}(e_i))\). An edge’s cable must support the current load of its destination. Hence we might define the edge’s current load as \(I_{\text{ave}}(e_i) = I_{\text{ave}}(\text{Tail}(e_i))\) and \(I_{\text{max}}(e_i) = I_{\text{max}}(\text{Tail}(e_i))\). We will denote an edge between nodes \(n_i\) and \(n_j\) by \(e(n_i, n_j)\).

The cable used along an edge is constrained by

\[ I_{\text{max}}(e_i) \leq I(\text{Cable}(e_i)) \]  
...Constraint 1 (C1)

The cable cost along an edge:

\[ \text{CableCost}(e_i) = \text{Len}(e_i) \cdot \text{Cost}(\text{Cable}(e_i)) \]  
...Eq. 4

The total cable cost is:

\[ \text{TotalCableCost} = \sum_{e \in k} \text{CableCost}(e_i) \]  
...Optimization Goal 1 (OG1)

The cost due to energy loss along an edge:

\[ \text{EnergyLossCost}(e_i) = 3 \cdot I_{\text{ave}}(e_i)^2 \cdot \text{Len}(e_i) \cdot \text{R}(\text{Cable}(e_i)) \cdot \text{UnitCost} \cdot \text{Life} \]  
...Eq. 5

where \(\text{UnitCost}\) is the unit cost of energy (in Rs\(\text{W}^{-1}\text{h}^{-1}\)) and \(\text{Life}\) is the working lifetime of the cable (in \(s\)). That is, for energy loss calculations we assume that the cable has been working at \(I_{\text{ave}}(e_i)\) for a period \(\text{Life}\). The “3” implies three phases. In calculating the energy loss cost over the life of the cable network, we have, to keep the presentation simple, assumed that the average inflation rate and the discounting rate are the same. Even if the latter does not apply, incorporating it into our analysis is quite straightforward.

The total energy loss cost is:

\[ \text{TotalEnergyLossCost} = \sum_{e \in E} \text{EnergyLossCost}(e_i) \]  
...Optimization Goal 2 (OG2)

The voltage drop along an edge:

\[ \text{VoltageDrop}(e_i) = I_{\text{ave}}(e_i) \sqrt{[\text{D}(\text{Cable}(e_i))^2 + \text{R}(\text{Cable}(e_i))^2]} \cdot \text{Len}(e_i) \]  
...Eq. 6

The voltage drop along a path is the sum of the drops along its edges:

\[ \text{VoltageDrop}(\text{Path}) = \sum_{e \in \text{Path}} \text{VoltageDrop}(e_i) \]  
...Eq. 7

The maximum voltage drop on any path must not exceed some preset value:

\[ \text{MaxVoltageDrop} \geq \text{max} \cdot \text{VoltageDrop} (\text{Path}) \]  
...Constraint 2 (C2)
Finally, for obvious practical reasons the position of the PDB depends on the position of the appliances (Usually, it is housed in the same room as the appliances). The exact nature of this constraint varies, but we will assume that the position of the PDB is constrained by the positions of appliances situated at the most extreme ends. That is,

\[
\begin{align*}
\min X(n_i) & \leq X(n_{PDB}) \leq \max X(n_i) \\
\min Y(n_i) & \leq Y(n_{PDB}) \leq \max Y(n_i) \\
\end{align*}
\]

...Constraint 3 (C3)

The aim of the problem is to minimize optimizing goals OG1 and OG2 subject to constraints C1, C2 and C3 by varying the values of optimizing variables OV1 and OV2. That is, to minimize cable cost and energy loss, subject to voltage drop, PDB position limits, and current carrying capacity constraints, by changing the position of the PDB and the selected cables for the edges.

2.2. Approaches used so far

A1: Current Carrying Capacity only

This is the simplest method, and also probably the most widely used. The position of PDB is based on ambient suitability and convenience, and is bounded by the walls of the room or C3. The cables are initially selected according to C1. That is, the smallest (hence cheapest and one with the least current carrying capacity) cable that can bear the edges current load is selected for each of the edges. Next, these initial cables might be replaced with larger cables to satisfy the same conditions. This is usually done at the leaf end of the path along which the voltage drop is too large.

The main problem with this method is that the PDB is positioned somewhat arbitrarily. Hence, OV1 is not considered. Secondly, even if (by chance, though highly unlikely) the PDB is in the optimal position, the optimality of the selection of OV2, might be negated by C2. Finally, this technique does not consider OG2.

A2: The Median Method

An improvement on A1 is to position the PDB according to a method known as the “median method”.

\[
\begin{align*}
TotalCost &= \sum_{e \in E} Cost(e) \\
&= \sum_{e \in E} Len(e), Cost(Cable(e)) \\
&= \sum_{e \in E} [X(n_{PDB}) - X(n_i)] Cost(Cable(e)) \\
&\quad + \sum_{e \in E} [Y(n_{PDB}) - Y(n_i)] Cost(Cable(e)) \\
&\quad + 2h \sum_{e \in E} Cost(Cable(e)) \\
\end{align*}
\]

...Eq. 8

Let us consider \(X(n_{PDB})\) (independently of \(Y(n_{PDB})\)):

\[
\begin{align*}
TotalCost &= \sum_{i=0}^{k} [X(n_{PDB}) - X(n_i)] Cost(Cable(e_i)) \\
&= \sum_{i=0}^{k} \left| X(n_{PDB}) - X(n_i) \right| Cost(Cable(e_i)) \\
\end{align*}
\]

...Eq. 9

This has the form:

\[
y = \sum_{i=0}^{k} |x - a_i| b_i
\]

...Eq. 10

Let \(p\) be a permutation on \{0,1,...,k\} such that, \(a_{p_1} \leq a_{p_2} \leq ... \leq a_{p_k}\). For any \(x\) such that \(a_{p_r} < x < a_{p_{r+1}}\), there is a unique \(r \in \{0,1,...,k-1\}\) such that, \(a_{p_r} < x < a_{p_{r+1}}\).

If \(x\) is increased by a small amount \(\Delta x\), the change in \(y\),

\[
\Delta y = \sum_{i=0}^{r} \Delta x b_{p_i} - \sum_{i=r+1}^{k} \Delta x b_{p_i}
\]

...Eq. 11

As \(x\) is increased from \(a_{p_r}\) to \(a_{p_{r+1}}\). \(\Delta y\) is initially negative, then reduces, then becomes positive and then increases. Hence, to minimize \(y\) we have to select \(x\) such that \(\Delta y\) is closest to zero. That is, \(\sum_{i=0}^{r} \Delta x b_{p_i} = \sum_{i=r+1}^{k} \Delta x b_{p_i}\). This is the cumulative median of \(b_{p_i}\).

Hence, in this problem to find the optimal \(X(n_{PDB})\), we find the cumulative median point of \(Cost(Cable(e_i))\) ranked by \(X(n_i)\). This method varies OV1 to minimize OG1. Similarly, \(Y(n_{PDB})\) is found.

The OV2 are initially selected according to C1. Then, we apply the median method on OV1. Finally, we might have to reset OV2 to satisfy C2 and reset OV1 to satisfy C3.

This method is a significant improvement on A1; the PDB is no longer positioned taking only convenience into account. However, as before, the optimality of the selection of OV2, might be negated by C2 and C3. Finally, this technique too does not consider OG2.

A2a: Extended Median Method

The problem of OG2 not being considered in A1 can be addressed easily, by simply incorporating it into the median method.

To find the optimal \(X(n_{PDB})\) that optimizes both OG1 and OG2, we find the cumulative median point of \(Cost(Cable(e_i))\) ranked by \(X(n_i)\). The other steps are identical.
General problems with the above approaches and how they can be solved

If we did not have the voltage drop constraint (C2), then A2a would result in an optimal solution. However, due to the non-linear specifications of cables, it is difficult to incorporate the voltage drop factor into the approach. Hence, it cannot be optimal. Also, the median method based approaches can be applied only to topologies consisting a tree of depth two (that is MSB, one or more PDBs and appliances).

A suitable solution to this problem must be able to deal with both the above problems. That is, it must be able to deal with non-linear factors and be extensible to more complex topologies.  

2.3. A Different Approach: Simulated Annealing (SA)

The approach that we suggest is based on Simulated Annealing (SA). SA is a stochastic local search algorithm, commonly used for optimization problems. Simulated Annealing is based on an idea borrowed from thermodynamics [4].

Systems made of atoms (for example, crystals) have an associated energy. In theory, if the atoms are ordered in a certain “perfect” way, this energy is minimized. In such a case, the system forms a “pure crystal”, and is made up of millions of atoms lined up in order in all directions.

Interestingly, nature provides a very simple method of achieving this perfect minimal energy system. At high temperatures, molecules of a liquid move fairly freely relative to each other. If the liquid is cooled, this “thermal mobility” is lost. If the system at high temperature is slowly cooled, atoms have ample time to redistribute as they gradually lose their mobility. This enables them to form a system with minimal (or nearly minimal) energy. This is known as annealing and is commonly used in producing hardened steel. Rapid cooling would result in a state with higher-than-minimal energy.

In any heated metal, the probability of some cluster of atoms at a position \( r \) exhibiting a specific energy state \( E(r) \) at some temperature \( T \) is defined by the Boltzmann probability factor:

\[
P(E(r)) = e^{-\frac{E(r)}{k_B T}} \quad \text{...Eq. 12}
\]

where \( k_B \) is the Boltzmann constant.

We can apply this idea to a standard search problem as follows. We can define an objective function \( f(\bar{x}) \) where \( \bar{x} \) are various design variables in the current state, for the search that is minimized in the optimal case. We replace \( E(r) \) with \( f(\bar{x}) \). We also need to replace \( T \) (temperature) with some suitable value. Since, the system cools with time, this is typically a decreasing function of time (say \( g(t) \)). \( k_B \) is also replaced with some suitable constant (say \( C \)).

In standard iterative improvement methods (for example hill climbing), a set of neighbours is generated until one which results in an improvement in the objective function is noted; in which case the trial point is accepted. However, since this allows movement in only one direction, it cannot escape local optima. In the case of annealing, a transition from a low energy state to a high energy state is possible, even though the probability of vice versa (that is, high to low) is higher. Hence, by modifying standard iterative improvement methods to simulate annealing we have a mechanism of escaping local optima.

This may be done as follows. As before, a set of neighbours is generated. If a neighbour results in an improvement in the objective function, it is accepted directly, as usual (in fact it is probabilistically accepted with probability of one). However, if one neighbour (with design variables \( \bar{x} \)) results in a deterioration in the objective function (from the initial value \( f(\bar{x}) \) to \( f(\bar{x}') \)), it still accepts it with probability:

\[
P(\bar{x}') = e^{\frac{f(\bar{x}) - f(\bar{x}')}{-k_B T}} \quad \text{...Eq. 13}
\]

This makes SA a stochastic search method.

Ascertaining whether a solution is “optimal”

One sure way of finding the optimal solution is to use a brute force search. However this results in a very large number of possibilities. For example, if \( X(n_{\text{pdb}}) \) and \( Y(n_{\text{pdb}}) \) are restricted to integer values in 0,1,...,50 and if we have 15 appliances and 15 different cable types, we have over \( 10^{15} \) possibilities. The search can be improved somewhat by considering C3, but these numbers are still too expensive. Alternatively, we could do a random search on the search space. But this is too complicated.

In many cases, it is impossible to say for sure what the optimal solution is. Hence, in this paper we will judge our solution to the problem based on comparisons with conventional methods.

3. Our Approach (A3)

3.1. Initial state

The initial state consists of an arbitrary state (not necessarily optimal) that may or may not satisfy C1, C2 and C3.

3.2. Generating Neighbours (New States)

Given a state, its neighbour states (required for the search) are generated by a successor function. The successor function modifies the position of the PDB (OV1) and the cables (OV2) by small, random increments, probabilistically.
This leads to a new state. The new state might (or might not) replace the current state as the next current state according to a utility function.

3.3. The Objective Function

The objective function is a real valued function that rewards low cable costs and low energy loss costs, and penalizes the failure to satisfy constraints on current carrying capacity, voltage drop and the practical location of the PDB.

\[ f(\tau) = (\text{TotalCable Cost} + \text{TotalEnergy Loss Cost}) \times \text{Pen} \]

...Eq.14

The penalty is calculated as follows: \( \text{Pen} = 2^v \) where \( v \) is the number of constraints out of C1, C2 and C3 that have been violated.

3.4. The Annealing Schedule

The annealing schedule is described in the following pseudo code.

```
Initialize the current state
Initialize the cables to the minimum cable size
Let \( \text{gen} = 100000 \)
For \( g = 0 \) to \( \text{gen} \) generations,
    Generate a new state, from the current state, via the successor function
    If \( f(\text{new state}) < f(\text{current state}) \) then
        current state = new state
    Else
        Let \( P(\tau) = e^{-\frac{[f(\text{new state})-f(\text{current state})]}{c \times g(t)}} \)
        where \( c = 100000 \)
        \( g(t) = k^t \) with \( k = 0.95 \)
        Set current state = new state
        with probability \( P(\tau) \)
    End If
End For
```

The choices for \( \text{gen} \), \( c \) and \( k \) might change from situation to situation. A reduction in \( \text{gen} \) reduces the probability of reaching an optimal solution. On the other hand, increasing \( \text{gen} \) slows down the algorithm. However, because the algorithm is relatively simple and involves straightforward calculations, even for large \( \text{gen} \), the running time is not too high. For example, for \( \text{gen} = 1,000,000 \), the running time is less than 20s. However, for the scenarios considered here \( \text{gen} = 100000 \) nearly always guarantees a very good solution. Even, smaller values might do so. The best choice for \( c \) depends on the scale of \( f(\tau) \). If costs take high values then \( c \) should be higher.

Since \( k^t \) is a decreasing, positive function with respect to \( g \), \( 0 < k < 1 \). The value of \( k \) controls the cooling rate. If \( k \) is close to unity then the cooling will be slow. If \( k \) is close to zero then the cooling will be fast. Note, that if \( k = 0 \), the algorithm mimics a standard random hill climb, without any annealing.

4. Results, Analysis and Interpretations

4.1. Assumptions

We assume that the cable installation has a life of 30 years, and operates 8 hours a day for 250 days a year.

These are all fairly conservative values (for instance, many factories operate two or three shifts). We also assume that the Unit cost of energy (UnitCost) is 10 Rupees (Sri Lanka) per kWh.

The maximum voltage drop on any path must not exceed 4% of 400V or 16V (MaxVoltageDrop).

We assumed that LoadFactor = 50% and DiversityFactor = 70%. The 50% load factor takes into account, a) precedence relationships associated with the loads and, b) operation at partial load. The 70% diversity factor takes into account the effect of precedence relationships on the maximum current drawn from the MSB.

As mentioned before, we assume three phase alternating current.

4.2. Examples Scenario

Obviously, there are an infinite number of scenarios which can be used to described the approach presented in this paper. We will analyze the results of one such scenario based on a typical industrial installation.

```
<table>
<thead>
<tr>
<th>( X(n_i) ) (in m)</th>
<th>( Y(n_i) ) (in m)</th>
<th>( I_{max}(n_i) ) (in A)</th>
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Table 1. Example Installation

We have assumed that the vertical height of the environment, \( h = 5m \).
The installation we are going to consider is a particular area in a factory. The positions of the main switch board (MSB) and the appliances are given relative to a selected coordinate point. The machines are three-phase motors absorbing the stated currents. The currents correspond to the rated motor currents. For this example, the current carrying capacity, resistance and reactance values are based on BS7671:2001, for multicore armoured thermoplastic pvc insulated cables laid on a perforated horizontal or vertical cable tray. The wiring costs have been provided by a popular Sri Lankan electrical contractor.

We have applied our approach (A3) to the problem and the other approaches currently in use (A1 and A2). A summary of the results is given below.

### Table 2. Cable Properties

<table>
<thead>
<tr>
<th>Cables Sizes (in $mm^2$)</th>
<th>$I(c_j)$ (in A)</th>
<th>$R(c_j)$ (mVA m$^{-1}$)</th>
<th>$D(c_j)$ (mVA m$^{-1}$)</th>
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</tr>
<tr>
<td>120</td>
<td>290</td>
<td>0.330</td>
<td>0.330</td>
<td>11392</td>
</tr>
<tr>
<td>150</td>
<td>332</td>
<td>0.260</td>
<td>0.260</td>
<td>13740</td>
</tr>
<tr>
<td>185</td>
<td>378</td>
<td>0.210</td>
<td>0.210</td>
<td>18096</td>
</tr>
<tr>
<td>240</td>
<td>445</td>
<td>0.165</td>
<td>0.165</td>
<td>21170</td>
</tr>
<tr>
<td>300</td>
<td>510</td>
<td>0.135</td>
<td>0.135</td>
<td>27110</td>
</tr>
<tr>
<td>400</td>
<td>590</td>
<td>0.100</td>
<td>0.100</td>
<td>36158</td>
</tr>
</tbody>
</table>

The actual cost reduction in A3 is wholly due to the reduction in energy loss cost. This has been achieved by using larger cables (in addition to efficiently positioning the PDB), and as a result, the cable cost has actually increased.

It follows that A3 will be better suited to scenarios where energy loss costs are more significant relative to cable costs. For example, if the installation has a longer life span or if the unit cost of energy is higher, A3 is likely to perform better. However, we see that A3 provides fair savings even in cases where the energy loss cost plays a lesser role. Quoted below are the cost reductions in A2 and A3 relative to A1 in several such scenarios.

### Table 4. Percentage cost reduction in A2 and A3 relative to A1

<table>
<thead>
<tr>
<th>Life Span (years)</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit Energy Cost (Rs/kWh)</td>
<td>8</td>
<td>8</td>
<td>8</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>A2</td>
<td>20%</td>
<td>17%</td>
<td>16%</td>
<td>20%</td>
<td>18%</td>
<td>16%</td>
</tr>
<tr>
<td>A3</td>
<td>23%</td>
<td>25%</td>
<td>31%</td>
<td>23%</td>
<td>30%</td>
<td>34%</td>
</tr>
</tbody>
</table>

Listed next are the sizes of cables from the PDB, to the MSB and to the appliances from solutions obtained by A1, A2 and A3.

### Table 5. Cable Sizes

<table>
<thead>
<tr>
<th>Cables Sizes (mm$^2$)</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSB</td>
<td>185</td>
<td>185</td>
<td>240</td>
</tr>
<tr>
<td>L1</td>
<td>10</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>L2</td>
<td>10</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>L3</td>
<td>10</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>L4</td>
<td>6</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>L5</td>
<td>4</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>L6</td>
<td>6</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>L7</td>
<td>6</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>L8</td>
<td>6</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>L9</td>
<td>1.5</td>
<td>2.5</td>
<td>6</td>
</tr>
<tr>
<td>L10</td>
<td>2.5</td>
<td>2.5</td>
<td>6</td>
</tr>
<tr>
<td>L11</td>
<td>2.5</td>
<td>2.5</td>
<td>6</td>
</tr>
<tr>
<td>L12</td>
<td>1.5</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>L13</td>
<td>1.5</td>
<td>1.5</td>
<td>2.5</td>
</tr>
<tr>
<td>L14</td>
<td>6</td>
<td>6</td>
<td>25</td>
</tr>
<tr>
<td>L15</td>
<td>6</td>
<td>10</td>
<td>25</td>
</tr>
<tr>
<td>L16</td>
<td>6</td>
<td>10</td>
<td>25</td>
</tr>
</tbody>
</table>

Under fairly simple scenarios (as the one discussed) with only a single layer of distribution boards between the MSB and the appliances, A1 and A2 are able to obtain a fairly good minimization of cable cost. Hence, as mentioned it is through minimizing energy loss cost that A3 gains an advantage. This is apparent from the cable sizes: A3 uses much larger cables.
4.3. Justification of Choice of Approach

Incorporating practical constraints
The main advantage in the stated approach is that it can easily incorporate constraints. Adding constraints to conventional techniques require fairly complex changes to the underlying mathematics. Also, it might not be possible to incorporate certain constraints.

As far as this technique is concerned adding constraints does not drastically change anything. As we saw, the number of constraints simply defines a simple penalty function.

The ability to incorporate constraints is very important in practical situations. In fact, these might vary from scenario to scenario. We might have to incorporate various aesthetic and building constraints. For example the positions of the PDBs might be constrained to a set of columns located in the room with the appliances.

Expansion
In order to compare the stated approach with the conventional approach, we have limited our analysis to the simple one MSB and one PDB scenario. However, as was the case with constraints, this method can be easily extended to optimize more complex scenarios (for example, those with multiple PDBs or those with several power sources).

This is an important advantage of the stated approach since many of the other approaches (e.g. median based techniques) cannot be used for more complex environments.

Scalability
The stated technique can be easily be scaled to incorporate priorities on various conflicting factors. For example, parameters can be changed to tradeoff optimality versus running time. In most cases, it is possible to find optimal solutions extremely fast – hence, a this type of tradeoff might not even be necessary. Other competing factors can also be considered relative to one another, for example cable cost and energy loss cost.

4.4. Possible future work

Complexity of Scenarios
To enable comparisons with the results of the conventional method and other methods, we have restricted our analysis to fairly simple scenarios. However, as mentioned, the approach can deal with far more complex environments. These might be explored in future work.

Expressing the solution in a mathematically rigorous form
We have not justified the results from a mathematical point of view. Any work in this direction is likely to enhance understanding of our approach, and would probably make its validity more acceptable.

5. Conclusion
We have presented a new approach to the sizing of low voltage cables and positioning of distribution switchgear, in industrial and commercial end-user environments. The approach aims at minimizing both the sum of cable costs and the energy-loss costs subject to voltage drop, current carrying capacity, location and other constraints. Several experimental scenarios indicate that our approach could enable significant reduction in overall costs.

References