Final Exam  
(100 points)

**Honor Code**: In contrast to the homework assignments, you may **not** collaborate on this midterm exam. You may not discuss the exam with anybody but the TAs and the instructor, who will only answer clarification questions. You may use no books other than the Enderton textbook and the handouts on provability logic.

1. (10 points) Enderton Section 3.5 #2 (pg. 245)

2. (15 points) Enderton Section 3.6 #3 (pg. 264)

3. (10 points) Enderton Section 3.6 #9 (pg. 264) *(Note: The proof here is an application of Rice’s Theorem (Theorem 36J) which we did not go over in class. So, the point of the problem is to first read pages 258 - 260, understand Rice’s Theorem then use it to solve this problem.)*

4. (10 points) Enderton Section 3.6 #13 (pg. 265)

5. (20 points) Enderton Section 3.6 #14 (pg. 265)

6. (10 points) Enderton Section 3.6 #15 (pg. 265)

7. (10 points) Enderton Section 3.7 #1 (pg. 275)

8. (15 points) From *Computability and Logic* by Boolos, Burgess and Jeffrey, pg. 336, 337

   A sentence $A$ is **modalized** in the sentence letter $p$ if every occurrence of $p$ in $A$ is part of a subsentence beginning with ‘$\Box$’ (note that sentence letters not containing $p$ are vacuously modalized in $p$). A sentence $H$ is a **fixed-point** of $A$ with respect to $p$ if $H$ contains only sentence letters contained in $A$, $H$ does not contain $p$ and

   $$\vdash_{GL} [(p \leftrightarrow A) \land \Box(p \leftrightarrow A)] \rightarrow (p \leftrightarrow H)$$

   **Theorem** (Fixed point theorem). If $A$ is modalized in $P$, then there exists a fixed point $H$ for $A$ relative to $p$.

   Verify (i.e., provide the required derivation) the following fixed points (this is half of table 27-1 on pg. 337):

   $$\begin{align*}
   A & \quad \Box p \quad \neg \Box p \quad \Box \neg p \\
   H & \quad \top \quad \neg \bot \quad \Box \bot
   \end{align*}$$

The midterm is DUE June 10, 2009 at NOON.